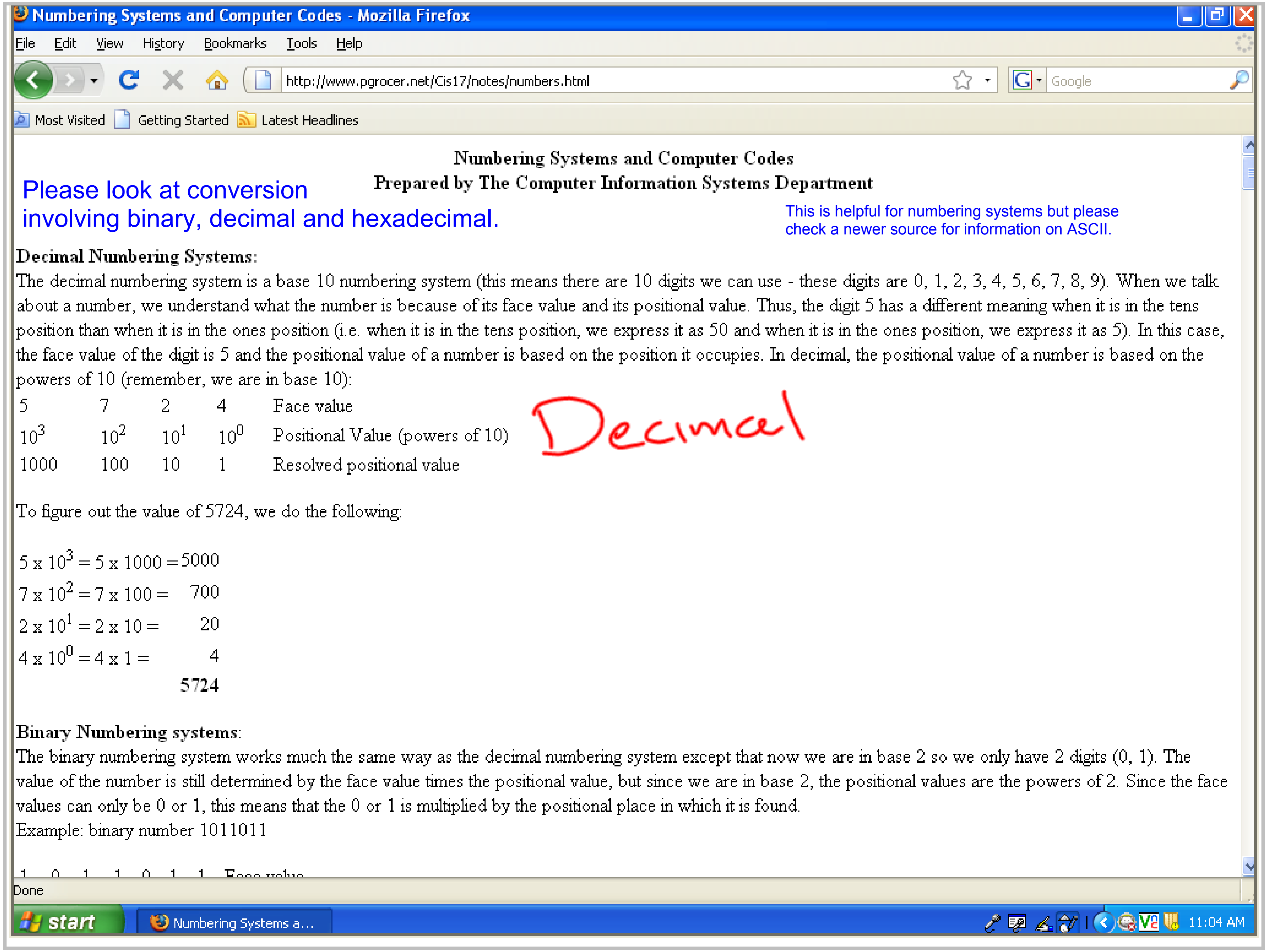

Numbering Systems and Computer Codes

Prepared by The Computer Information Systems Department

Please look at conversion involving binary, decimal and hexadecimal.

This is helpful for numbering systems but please check a newer source for information on ASCII.

Decimal Numbering Systems:

The decimal numbering system is a base 10 numbering system (this means there are 10 digits we can use - these digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). When we talk about a number, we understand what the number is because of its face value and its positional value. Thus, the digit 5 has a different meaning when it is in the tens position than when it is in the ones position (i.e. when it is in the tens position, we express it as 50 and when it is in the ones position, we express it as 5). In this case, the face value of the digit is 5 and the positional value of a number is based on the position it occupies. In decimal, the positional value of a number is based on the powers of 10 (remember, we are in base 10):

5	7	2	4	Face value
10^3	10^2	10^1	10^0	Positional Value (powers of 10)
1000	100	10	1	Resolved positional value

Decimal

To figure out the value of 5724, we do the following:

$5 \times 10^3 = 5 \times 1000 = 5000$

$7 \times 10^2 = 7 \times 100 = 700$

$2 \times 10^1 = 2 \times 10 = 20$

$4 \times 10^0 = 4 \times 1 = 4$

5724

Binary Numbering systems:

The binary numbering system works much the same way as the decimal numbering system except that now we are in base 2 so we only have 2 digits (0, 1). The value of the number is still determined by the face value times the positional value, but since we are in base 2, the positional values are the powers of 2. Since the face values can only be 0 or 1, this means that the 0 or 1 is multiplied by the positional place in which it is found.

Example: binary number 1011011

1 0 1 1 0 1 1 Face value

Binary – 2 Digits 0, 1

Base 2 system

1	0	1	1	Face Value
2^3	2^2	2^1	2^0	Positional
8	4	2	1	

$1 \times 2^3 = 1 \times 8 = 8$

$0 \times 2^2 = 0 \times 4 = 0$

$1 \times 2^1 = 1 \times 2 = 2$

$1 \times 2^0 = 1 \times 1 \quad 1$

$\overline{11}$

Binary	Decimal
0	0
1	1
10	2
11	3
100	4
101	5
110	6
111	7
1000	8
1001	9

This shows binary and the decimal equivalent.

The 2 and 1 are turned on so it is 3			1	1
The 2 is all that is turned on so it is 2			1	0
The 1 is all that is turned on so it is 1				1
	2^3	2^2	2^1	2^0
	8	4	2	1
The 4 is turned on so it is 4		1	0	0
The 4 and 1 are turned on so it is 5		1	0	1

10011101_2

The 128 has a 1 so it is on, the next one that is on is 16 and than 8, 4 and 1 which adds to 157

2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1
1	0	0	1	1	1	0	1

$157_{10} =$

128

29

-16

$13-8=5=4=1-1=0$

Now that I convert binary 10011101 from binary to decimal, I am converting it back to check. I can subtract 128 from 157 so I put a 1 on it because it is used. I have 29 left and I cannot subtract 64 or 32 from 29 so they are not used and I put 0 on them. I can subtract 16, 8 and 4 so they have a 1 on them. I cannot use 2 so it gets a 0 and I can use 1 so it gets a 1 on it.

128
16
8
4
1

157_{10}

1	0	1	0	0	0	1	0	1	1
512	256	128	64	32	16	8	4	2	1

$1010001011_2 = 651_{10}$

$$\begin{array}{r} 512 \\ 128 \\ 8 \\ 2 \\ 1 \\ \hline 651_{10} \end{array}$$

Converting from binary to decimal I multiply face value which is 1 or 0 by positional value which basically means if there is a 1 I add it in, if there is a 0 I do not since 0 times something is 0.

$345_{10} = 101011001_2$

$$\begin{array}{r} 345 \\ 256 \\ \hline 89 \\ -64 \\ \hline 25 \end{array}$$

$25 - 16 = 9 - 8 = 1$

Another conversion from binary to decimal or base 2 to base 10.

	1	0	1	0	1	1	0	0	1
2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
512	256	128	64	32	16	8	4	2	1

1011|1001

$1111_2 = 15_{10}$

8421

Hexadecimal is base 16 which means there are 16 digits:
0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F
That means any digit up through 15 can be represented with one digit.
That means that any four digit binary number can be represented with one hex digit.
Computer codes are typically
8 bits and breaking that in half
we have two 4 digit binary sets
that can each be represented with
one hex digit.

Hex 0 – 15

base 16

Binary	Decimal	Hex
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F
10000	16	10

__	__	__	__
2^3	2^2	2^1	2^0
8	4	2	1

$$1 \mid 0101 \mid 1101 \mid 1110 \mid 1011_2 = \underline{15DEB}_{16}$$

$$1 \quad 5 \quad D \quad E \quad B$$

Converting binary to hex and hex to binary.

$C7AF_{16}$ = binary

1100 0111 1010 1111

1	0	1	1
2^3	2^2	2^1	2^0
8	4	2	1

Addition in binary:
Remember that 1+ 1 is 10 + 1 is 11 + 1 is 100 etc

Check doc

$$\begin{array}{rrrrrl} & 1 & 1 & 1 & & \\ & 1 & 0 & 1 & 1 & — 11 \\ & & 1 & 1 & 1 & — 7 \\ + & & 1 & 0 & 1 & — 5 \\ \hline 1 & 0 & 1 & 1 & 1 & 23 \\ 16 & 8 & 4 & 2 & 1 & \end{array}$$

$$16+4+2+1=23$$

$$\begin{array}{r} 1 \\ +\ 1 \\ \hline 10 \\ +\ 1 \\ \hline 11 \\ +\ 1 \\ \hline 100 \end{array}$$

Dec

$$\begin{array}{rrrrrl}
 & 1 & 1 & 1 & & \\
 & 1 & 1 & 0 & 1 & - 13 \\
 & 1 & 1 & 1 & 0 & - 14 \\
+ & & & 1 & 1 & - 3 \\
\hline
1 & 1 & 1 & 1 & 0 & 30 \quad 11110 \\
2^4 & 2^3 & 2^2 & 2^1 & 2^1 & \\
16 & 8 & 4 & 2 & 1 &
\end{array}$$

$16+8+4+2=30$

$$\begin{array}{rr}
 & 1 \\
+ & 1 \\
\hline
1 & 0 \\
+ & 1 \\
\hline
1 & 1 \\
+ & 1 \\
\hline
10 & 0 \\
+ & 1 \\
\hline
10 & 1
\end{array}$$

$$\begin{array}{r}
{}^{10}\,{}^{11}\,{}^{10} \\
1011 \\
101 \\
11 \\
11 \\
+\quad 11 \\
\hline
11001
\end{array}$$

$$\begin{array}{r}
1 \\
+\ 1 \\
\hline
10 \\
+\ 1 \\
\hline
11 \\
+\ 1 \\
\hline
100 \\
+\ 1 \\
\hline
101
\end{array}
\qquad
\begin{array}{r}
101 \\
+\ 1 \\
\hline
110 \\
+\ 1 \\
\hline
111 \\
+\ 1 \\
\hline
1000
\end{array}$$

Converting decimal to hex and hex to decimal.
See the handout for more information.

$457_{10} = 1C9_{16}$

$$\begin{array}{r} 457 \\ -256 \\ \hline 201 \\ -192 \\ \hline 9 \end{array}$$

$10 \times 16 = 160$

$$\begin{array}{r} 160 \\ 32 \\ \hline 192 \end{array}$$

12×16

	1	C	9
16^3	16^2	16^1	16^0
4096	256	16	1

$1 \times 256 = 256$

$C \times 16 = 192$

$1 \times 9 = 9$

$$\begin{array}{r} 256 \\ 192 \\ 9 \\ \hline 457 \end{array}$$

Convert hex to dec

$1C9_{16} = \_\_\_\__{10}$