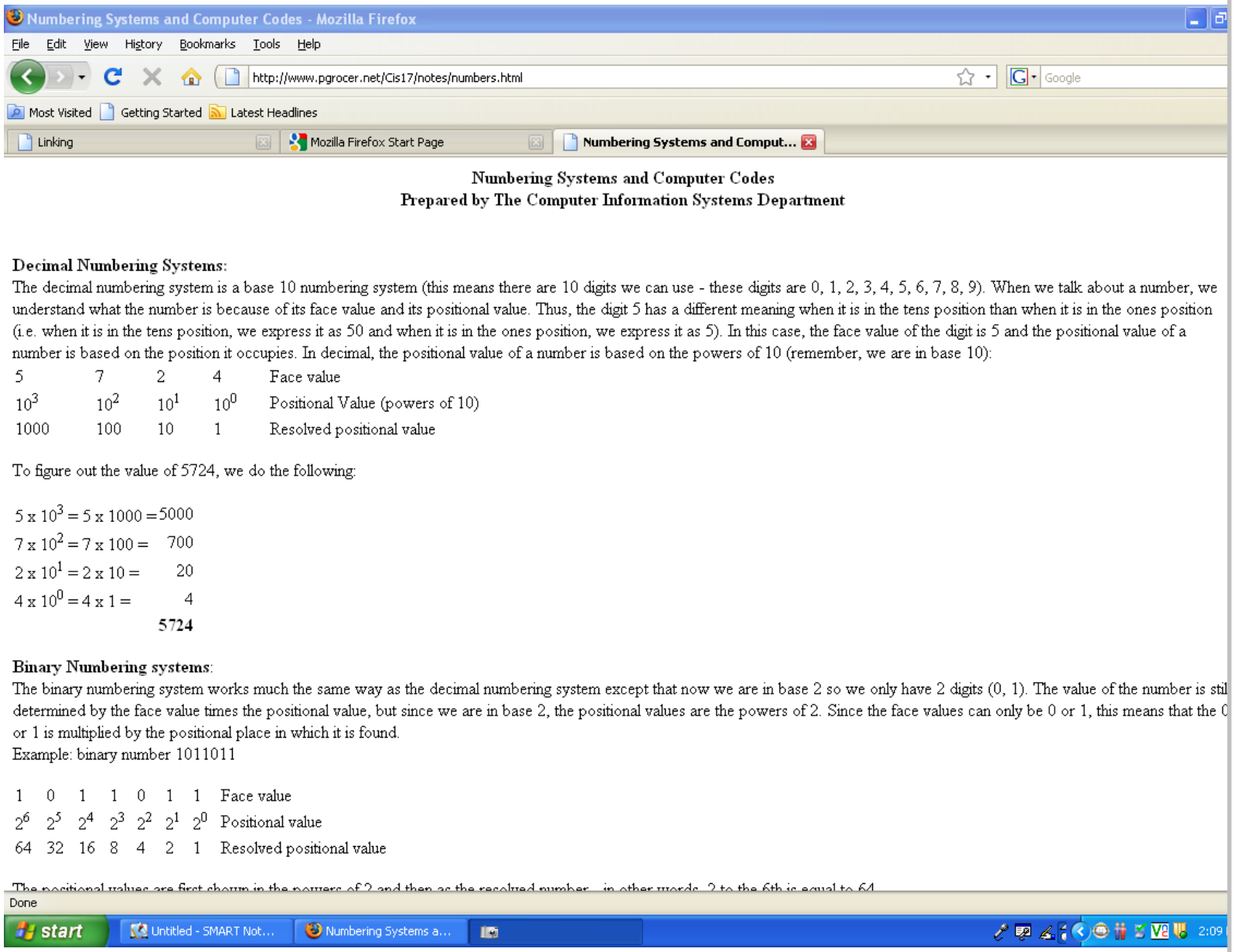


Numbering Systems and Computer Codes
Prepared by The Computer Information Systems Department

Decimal Numbering Systems:
The decimal numbering system is a base 10 numbering system (this means there are 10 digits we can use - these digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). When we talk about a number, we understand what the number is because of its face value and its positional value. Thus, the digit 5 has a different meaning when it is in the tens position than when it is in the ones position (i.e. when it is in the tens position, we express it as 50 and when it is in the ones position, we express it as 5). In this case, the face value of the digit is 5 and the positional value of a number is based on the position it occupies. In decimal, the positional value of a number is based on the powers of 10 (remember, we are in base 10):

5	7	2	4	Face value
10^3	10^2	10^1	10^0	Positional Value (powers of 10)
1000	100	10	1	Resolved positional value

To figure out the value of 5724, we do the following:

$5 \times 10^3 = 5 \times 1000 = 5000$
$7 \times 10^2 = 7 \times 100 = 700$
$2 \times 10^1 = 2 \times 10 = 20$
$4 \times 10^0 = 4 \times 1 = 4$
5724

Binary Numbering systems:
The binary numbering system works much the same way as the decimal numbering system except that now we are in base 2 so we only have 2 digits (0, 1). The value of the number is stil determined by the face value times the positional value, but since we are in base 2, the positional values are the powers of 2. Since the face values can only be 0 or 1, this means that the 0 or 1 is multiplied by the positional place in which it is found.
Example: binary number 1011011

1	0	1	1	0	1	1	Face value
2^6	2^5	2^4	2^3	2^2	2^1	2^0	Positional value
64	32	16	8	4	2	1	Resolved positional value

The positional values are first shown in the powers of 2 and then as the resolved number - in other words, 2 to the 6th is equal to 64

Decimal base 10 0-9

8	5	3
10^2	10^1	10^0
100	10	1

face value

positional value

$8 \times 10^2 + 5 \times 10^1 + 3 \times 1$

$800 + 50 + 3$

853

Binary base 2 (0,1)

face value $1\,0\,1\,1\,1\,0\,1_2 = \underline{\quad 93 \quad}_{10}$

positional value $2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

2^7 value $64\ 32\ 16\ 8\ 4\ 2\ 1$

128

$1 \times 64 = 64$
$0 \times 32 = 0$
$1 \times 16 = 16$
$1 \times 8 = 8$
$1 + 4 = 5$
$0 \times 2 = 0$
$1 \times 1 = 1$

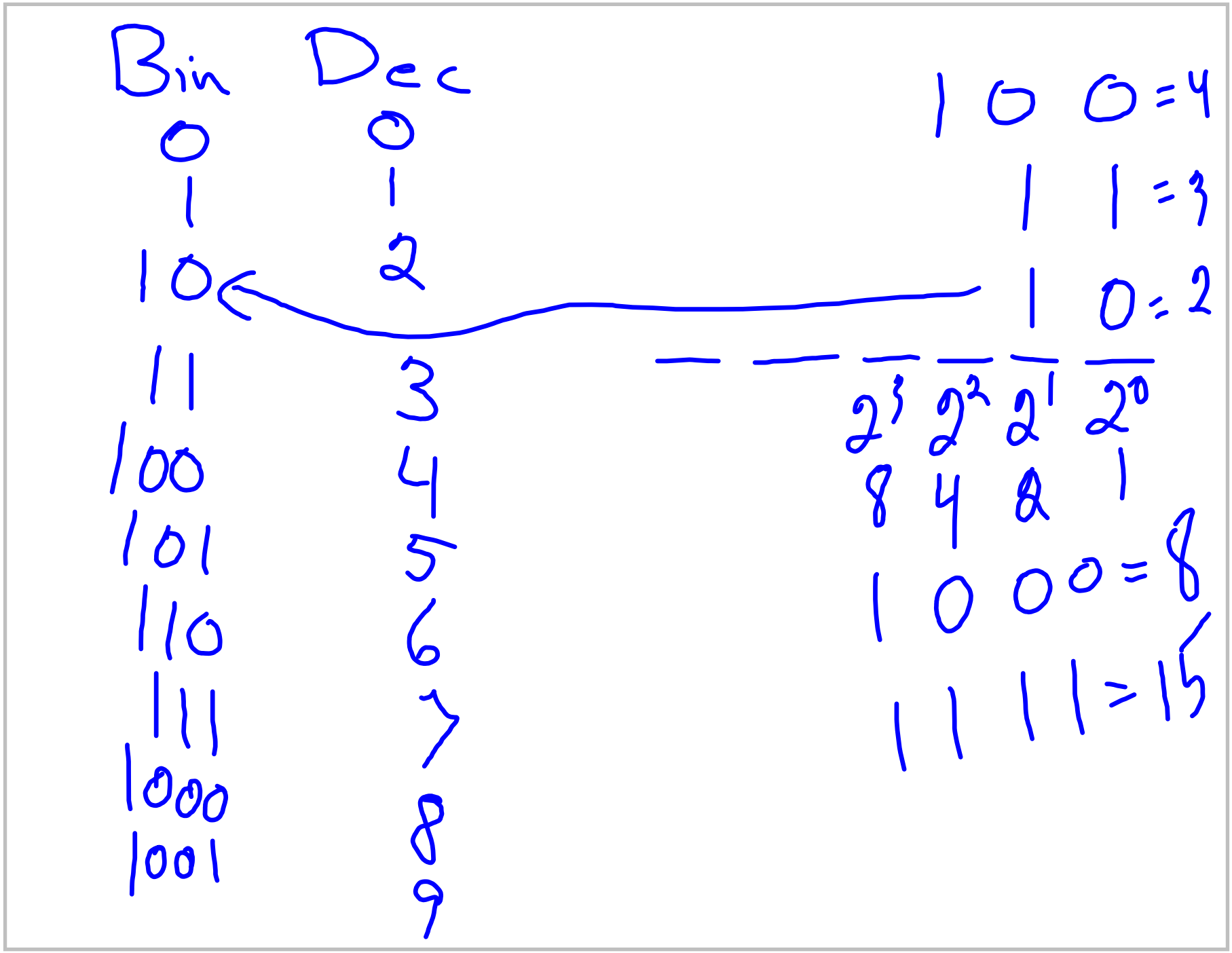
$93_{10} = \underline{\quad\quad}_2$

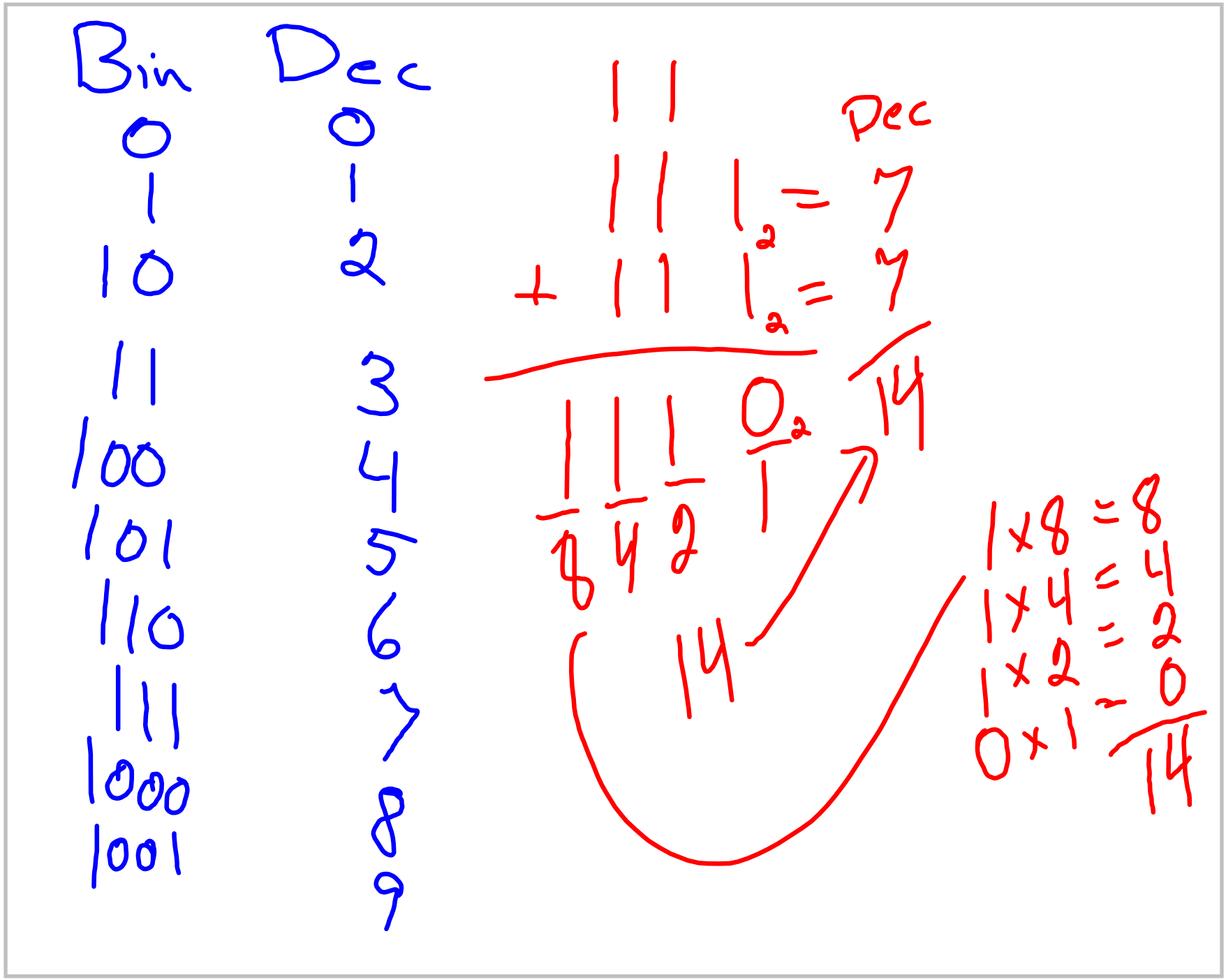
-64
29
-16
13
-8
5
-4
$1 - 1 = 0$

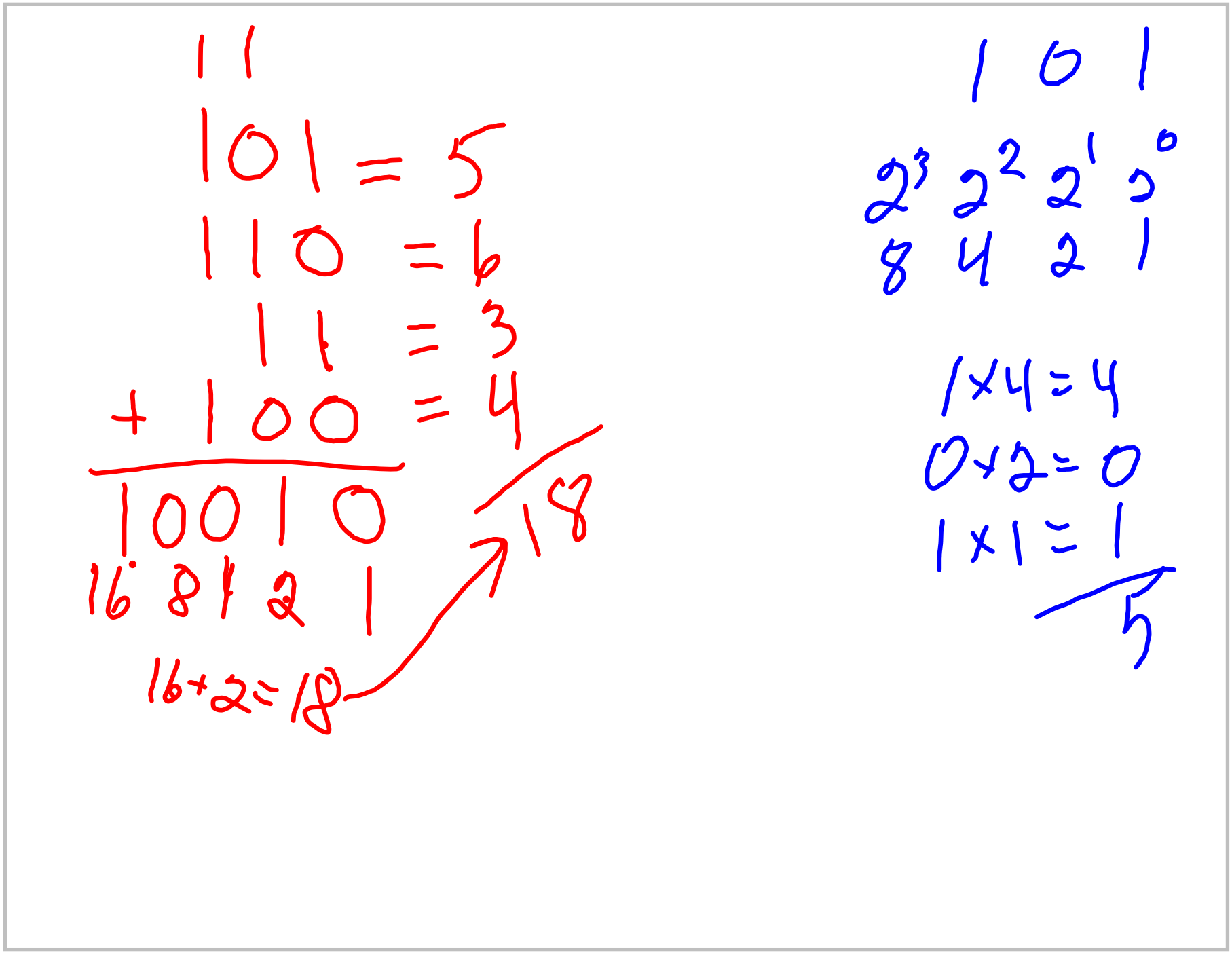
$1\ 0\ 1\ 1\ 1\ 0\ 1_2$

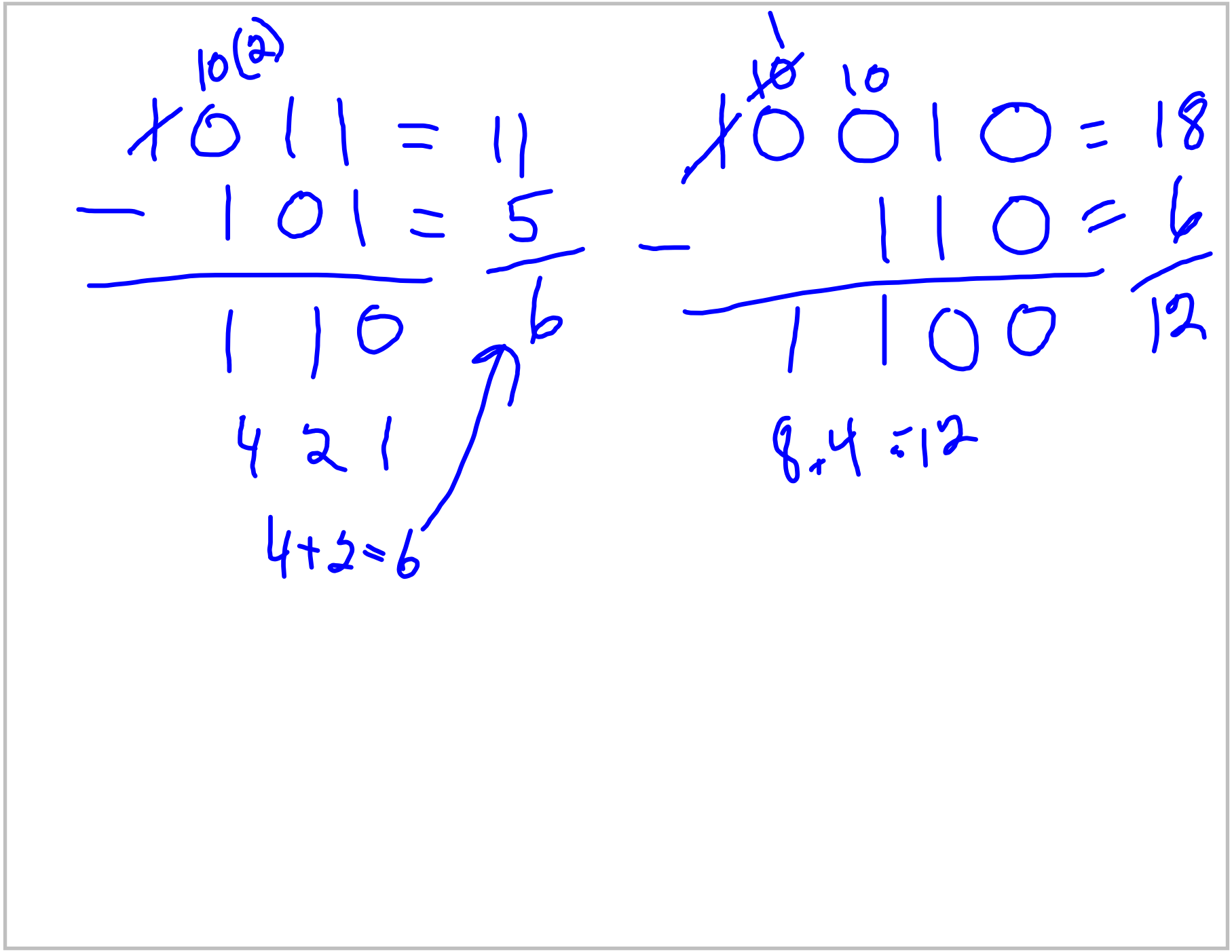
$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$64\ 32\ 16\ 8\ 4\ 2\ 1$


Bin
Dec
0
0
1
1
10
2
11
3
100
4
101
5
110
6
111
7
1000
8
1001
9
1 0 0 = 4
1 1 = 3
1 0 = 2
2^3 2^2 2^1 2^0
8 4 2 1
1 0 0 0 = 8
1 1 1 1 = 15


Bin
Dec
0
0
1
1
10
2
11
3
100
4
101
5
110
6
111
7
1000
8
1001
9
Dec
1 1
1 1 1 = 7
+ 1 1 1 = 7
1 1 1 0 = 14
1/8 1/4 1/2 1
14
1x8 = 8
1x4 = 4
1x2 = 2
0x1 = 0
14


11
101 = 5
110 = 6
11 = 3
+ 100 = 4
10010
16 8 4 2 1
16+2=18
18
1 0 1
2³ 2² 2¹ 2⁰
8 4 2 1
1×4=4
0×2=0
1×1=1
5


10(2)
1011 = 11
- 101 = 5
110
6
421
4+2=6
1
10 10
10010 = 18
- 110 = 6
1100
12
8+4=12

Hex 16 digits (0-9, A, B, C, D, E, F)

10 11 12 13 14 15

$$\underline{\quad}\;\underline{B}\;\underline{3}\;\underline{D}_{face} = \underline{2877}_{10}$$

16^3 16^2 16^1 16^0 positional

4096 256 16 1

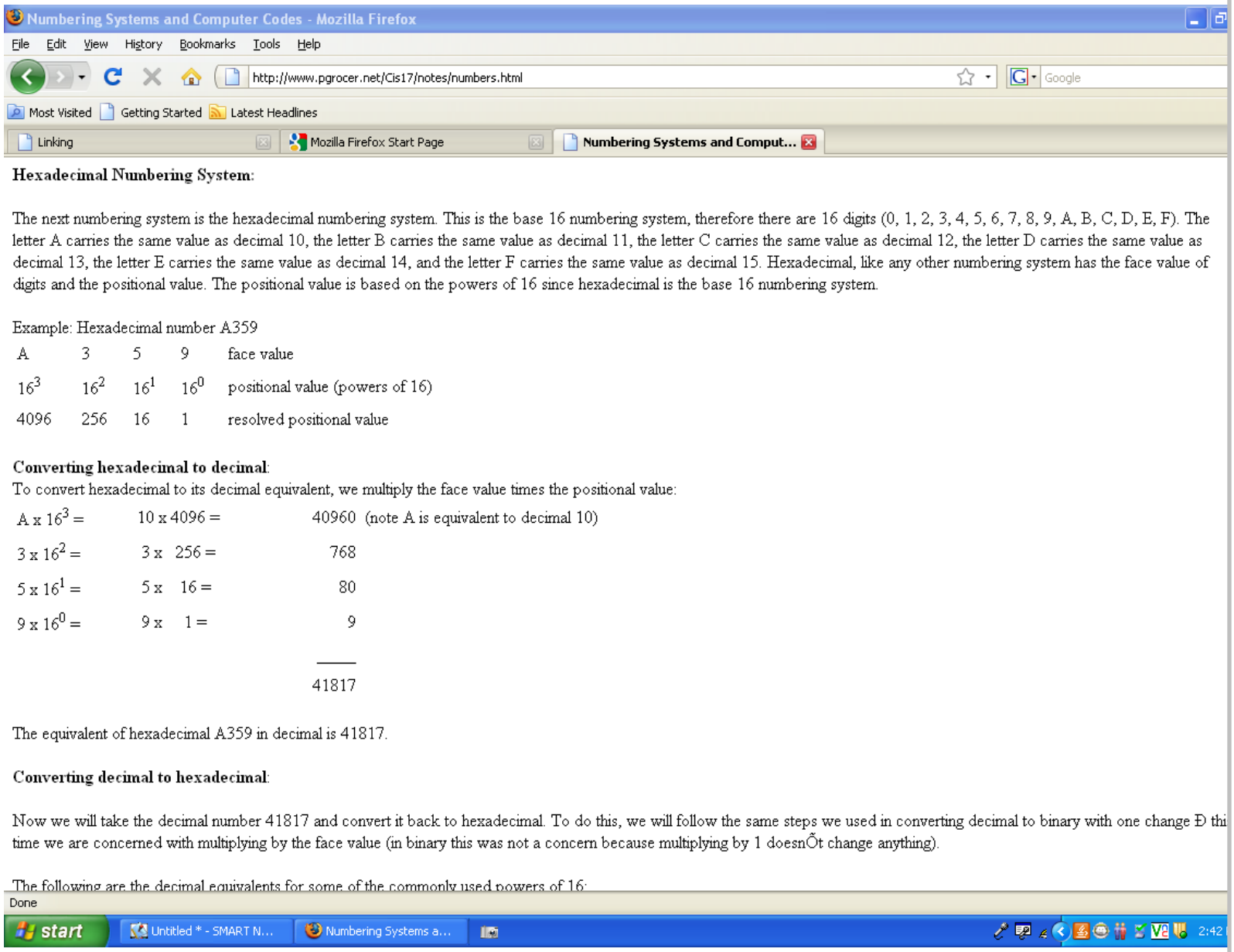
$B \times 16^2 = B \times 256 = 11 \times 256 = 2816$

$3 \times 16^1 = 3 \times 16 = 48$

$D \times 16^0 = D \times 1 = 13 \times 1 = 13$

2877

256 × 11 = 256 + 256 = 2816



Hexadecimal Numbering System:

The next numbering system is the hexadecimal numbering system. This is the base 16 numbering system, therefore there are 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). The letter A carries the same value as decimal 10, the letter B carries the same value as decimal 11, the letter C carries the same value as decimal 12, the letter D carries the same value as decimal 13, the letter E carries the same value as decimal 14, and the letter F carries the same value as decimal 15. Hexadecimal, like any other numbering system has the face value of digits and the positional value. The positional value is based on the powers of 16 since hexadecimal is the base 16 numbering system.

Example: Hexadecimal number A359

A	3	5	9	face value
16^3	16^2	16^1	16^0	positional value (powers of 16)
4096	256	16	1	resolved positional value

Converting hexadecimal to decimal:
To convert hexadecimal to its decimal equivalent, we multiply the face value times the positional value:

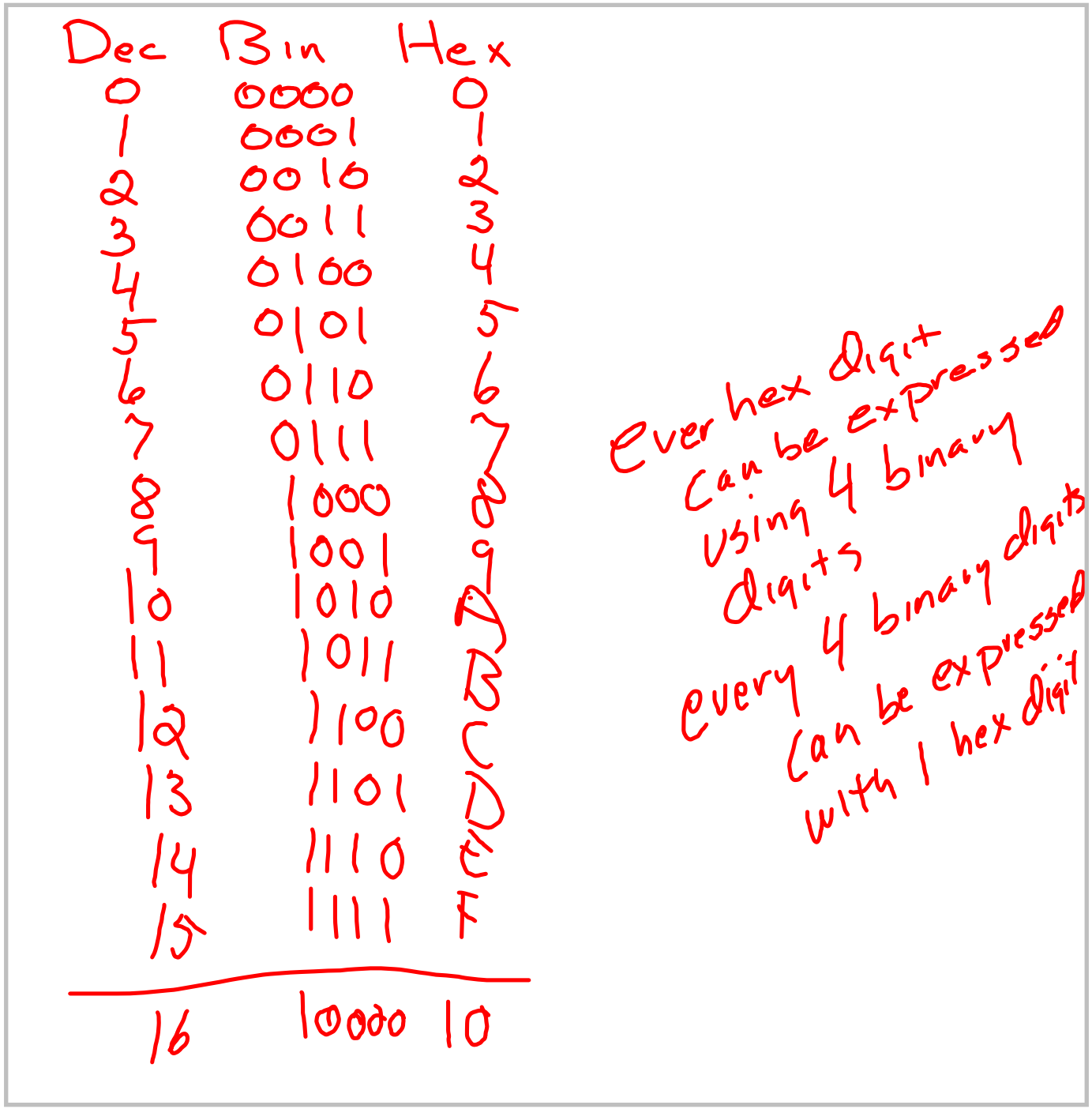
A x 16^3 =	10 x 4096 =	40960 (note A is equivalent to decimal 10)
3 x 16^2 =	3 x 256 =	768
5 x 16^1 =	5 x 16 =	80
9 x 16^0 =	9 x 1 =	9
		41817

The equivalent of hexadecimal A359 in decimal is 41817.

Converting decimal to hexadecimal:

Now we will take the decimal number 41817 and convert it back to hexadecimal. To do this, we will follow the same steps we used in converting decimal to binary with one change Ð thi time we are concerned with multiplying by the face value (in binary this was not a concern because multiplying by 1 doesnÕt change anything).

The following are the decimal equivalents for some of the commonly used powers of 16:


Dec Bin Hex
0 0000 0
1 0001 1
2 0010 2
3 0011 3
4 0100 4
5 0101 5
6 0110 6
7 0111 7
8 1000 8
9 1001 9
10 1010 A
11 1011 B
12 1100 C
13 1101 D
14 1110 E
15 1111 F
16 10000 10
Ever hex digit can be expressed using 4 binary digits
every 4 binary digits can be expressed with 1 hex digit

$B3D_{16} = 2877_{10} = \_\_\__{16}$

2877

2560 − 10

317 | 11

−256 − 1 | B

61

−48 − 3

13

$\_\_\_ = \underline{B}\,\underline{3}\,\underline{D}$

16^3 16^2 16^1 16^0

4096 256 16 1

Bin - hex

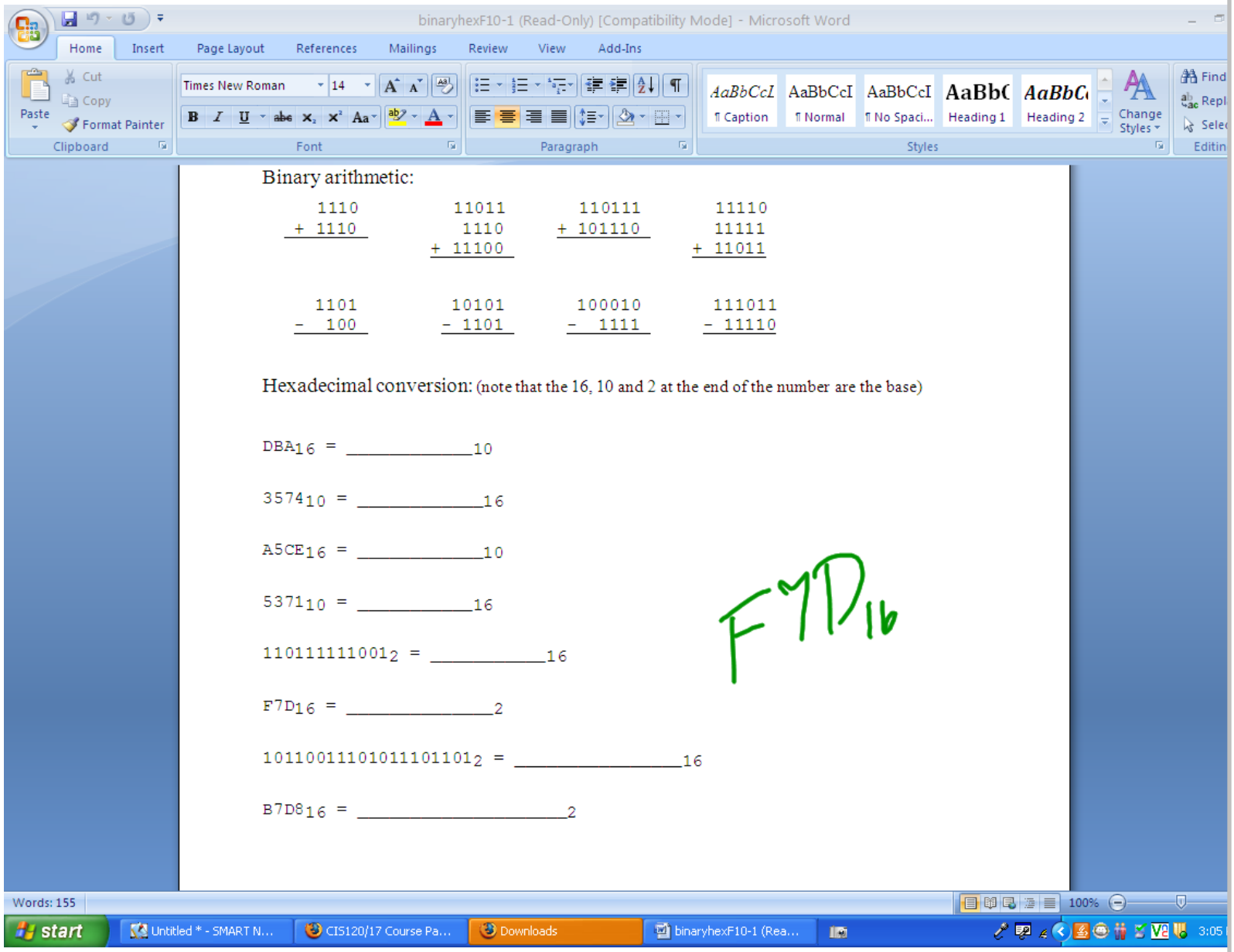
hex - bin

$$\underset{2}{0010}\ \underset{E}{1110}\ \underset{E}{1110}\ \underset{7}{0111}\ \underset{8}{1000}\ \underset{F}{1111}_2 = \underline{\qquad}_{16}$$

8421

$$AC9D2_{16} = \underline{10101100100111010010}_2$$

A → 1010, C → 1100, 9 → 1001, D → 1101, 2 → 0010



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